BASIC DIFFERENTIAL EQUATIONS

EXTRA CREDIT FOR MATH 122-01
DUE MARCH 27, 2008

The answers to the problems below should be presented neatly, (either typed or written **very** neatly).

You may discuss the problems with other students. However each student is responsible for the final preparation of his or her own paper.

A differential equation is an equation that contains a derivative within it. Differential equations often arise when studying applications of calculus because they allow you to explain mathematically how one variable is changing with respect to another. The first extra credit assignment dealt with a particular differential equation that described exponential population growth: $\frac{dP}{dt} = rP$. A more complicated example is:

$$\frac{d^3y}{dx^3} + \cos x \sin y \frac{dy}{dx} - e^x = 0.$$

A solution to a differential equation is a function $y = f(x)$ that satisfies the differential equation. The general solution to a differential equation is the set of all possible solutions to the differential equation. This project will focus on certain classes of differential equations that can be solved using basic techniques described in the textbook.

A differential equation is a **first-order linear equation** if it is of the following form:

$$\frac{dy}{dx} + p(x)y = q(x),$$

where $p(x)$ and $q(x)$ are continuous functions. Section 9.1 of the textbook describes how to solve differential equations of this form. You should read Section 9.1 and then answer the following questions.

Problem 1. *Find the general solution of the equation:*

$$\frac{dy}{dx} + 2xy = x.$$

Problem 2. *Find the general solution of the equation:*

$$\frac{dy}{dx} - e^x y = 0.$$

A differential equation is **separable** if it is of the following form:

$$p(x) + q(y)\frac{dy}{dx} = 0,$$

where $p(x)$ and $q(x)$ are continuous functions. Section 9.2 of the textbook describes how to solve differential equations of this form. You should read Section 9.2 and then answer the following questions.

Problem 3. *Find the general solution of the equation:*

$$\frac{dy}{dx} = xe^{y-x}.$$

Problem 4. *Find the general solution of the equation:*

$$\frac{dy}{dx} = y \sin{(2x+3)}.$$

A differential equation is a **homogeneous second-order linear differential equation with constant coefficients** if it is of the following form:

$$\frac{d^2y}{dx^2} + a\frac{dy}{dx} + by = 0,$$

where a and b are real numbers. Section 9.3 of the textbook describes how to solve differential equations of this form. You should read Section 9.3 and then answer the following questions.

Problem 5. *Find the general solution of the equation:*

$$\frac{d^2y}{dx^2} + 4\frac{dy}{dx} + 4y = 0.$$

Problem 6. *Find the general solution of the equation:*

$$\frac{d^2y}{dx^2} + 2\frac{dy}{dx} - 8y = 0.$$