1. Let \( x = (x_1, x_2, \ldots, x_n), y = (y_1, y_2, \ldots, y_n) \in \mathbb{R}^n \). Define 
\[
d^*(x, y) = |x_1 - y_1| + |x_2 - y_2| + \ldots + |x_n - y_n|.
\]
(a) Show that \( d^* \) is a metric on \( \mathbb{R}^n \).
(b) Draw the ball of radius 1 centered at the origin in \( \mathbb{R}^2 \) in this metric. That is \( B_{d^*}((0, 0), 1) \).
(c) Show that the topology induced by \( d^* \) on \( \mathbb{R}^2 \) is the same as the standard topology on \( \mathbb{R}^2 \).

2. Let \( x = (x_1, x_2, \ldots, x_n), y = (y_1, y_2, \ldots, y_n) \in \mathbb{R}^n \). Define 
\[
d^*(x, y) = \min_{1 \leq i \leq n} |x_i - y_i|.
\]
Is \( d^* \) a metric on \( \mathbb{R}^n \)? If so, prove it. If not, explain why not.

3. Let \( X \) be any set and let \( x, y \in X \). Define \( d(x, y) = 1 \) if \( x \neq y \) and \( d(x, x) = 0 \).
(a) Show that \( d \) is a metric on \( X \).
(b) What topology does \( d \) induce?