Written Homework Problems

Math 244

Due February 22, 2010

1. Let $\mathbf{x} = (x_1, x_2, \ldots,\ x_n), \mathbf{y} = (y_1, y_2, \ldots, y_n) \in \mathbb{R}^n$. Define

$$d^*(\mathbf{x}, \mathbf{y}) = |x_1 - y_1| + |x_2 - y_2| + \ldots + |x_n - y_n|.$$

   (a) Show that d^* is a metric on $\mathbb{R}^n$.

   (b) Draw the ball of radius 1 centered at the origin in $\mathbb{R}^2$ in this metric. That is $B_{d^*}((0,0),1)$. Draw the ball of radius 1 centered at the origin in $\mathbb{R}^3$ in this metric. That is $B_{d^*}((0,0,0),1)$.

   (c) Show that the topology induced by d^* on $\mathbb{R}^2$ is the same as the standard topology on $\mathbb{R}^2$.

2. Let $\mathbf{x} = (x_1, x_2, \ldots,\ x_n), \mathbf{y} = (y_1, y_2, \ldots, y_n) \in \mathbb{R}^n$. Define

$$d_*(\mathbf{x}, \mathbf{y}) = \min_{1 \le i \le n} |x_i - y_i|.$$

   Is d_* a metric on $\mathbb{R}^n$? If so, prove it. If not, explain why not.

3. Let X be any set and let $x, y \in X$. Define $d(x, y) = 1$ if $x \neq y$ and $d(x, x) = 0$.

   (a) Show that d is a metric on X.

   (b) What topology does d induce?