Written Homework Problems

Math 244

Due April 22, 2010

1. Consider the set $(\mathbb{R} - \{0\}) \cup \{0_a, 0_b\}$. We give a base for a topology on this set consisting of four types of subsets:

   - If $a < b < 0$, then the interval (a, b) is open.
   - If $0 < a < b$, then the interval (a, b) is open.
   - If $a < 0 < b$, then the set $((a, b) - \{0\}) \cup \{0_a\}$ is open.
   - If $a < 0 < b$, then the set $((a, b) - \{0\}) \cup \{0_b\}$ is open.

   The resulting topological space is called *the line with two origins.*

   Show that the line with two origins is locally homeomorphic to $\mathbb{R}$, but is not a 1-manifold because it is not Hausdorff.

2. A space is *second countable* if there is a countable base for the topology. Show that the real line with the discrete topology is locally homeomorphic to $\mathbb{R}^0$ and is Hausdorff, but is not a 0-manifold because it is not second countable. (Recall that the set of all real numbers is uncountable.)

3. Show that the product of an n-manifold and an m-manifold is an $(n + m)$-manifold. (Recall that the product of two countable sets is countable.)