1. Consider the set \((\mathbb{R} - \{0\}) \cup \{0_a, 0_b\}\). We give a base for a topology on this set consisting of four types of subsets:

- If \(a < b < 0\), then the interval \((a, b)\) is open.
- If \(0 < a < b\), then the interval \((a, b)\) is open.
- If \(a < 0 < b\), then the set \(((a, b) - \{0\}) \cup \{0_a\}\) is open.
- If \(a < 0 < b\), then the set \(((a, b) - \{0\}) \cup \{0_b\}\) is open.

The resulting topological space is called the line with two origins.

Show that the line with two origins is locally homeomorphic to \(\mathbb{R}\), but is not a 1-manifold because it is not Hausdorff.

2. A space is second countable if there is a countable base for the topology. Show that the real line with the discrete topology is locally homeomorphic to \(\mathbb{R}^0\) and is Hausdorff, but is not a 0-manifold because it is not second countable. (Recall that the set of all real numbers is uncountable.)

3. Show that the product of an \(n\)-manifold and an \(m\)-manifold is an \((n + m)\)-manifold. (Recall that the product of two countable sets is countable.)