

Problem Session Problems

Math 244

February 22, 2010

1. Show that $d(x, A) = 0$ if and only if $x \in \bar{A}$.
2. Let X be a metric space and let $x \in X$. Show that $\{x\}$ is closed in X .
3. A topological space X is **regular** if (one-pointed sets are closed and) for every $x \in X$ and every closed subset A of X , if $x \notin A$ there are disjoint open sets U and V in X such that $x \in U$ and $A \subset V$.

Show that metric spaces are regular.

4. A topological space X is **normal** if (one-pointed sets are closed and) for all disjoint closed subsets A and B of X , there are disjoint open sets U and V in X such that $A \subset U$ and $B \subset V$.

Show that metric spaces are normal.

5. Is there a metric on \mathbb{R} that induces the finite complement topology? If so, find it. If not, explain why.