Problem Session Problems

Math 244

February 8, 2010

1. Describe a collection of sets that are open in the standard topology on $\mathbb{R}$, but whose intersection is not open in the standard topology on $\mathbb{R}$.

2. Let $\mathcal{S} = \{B \subset \mathbb{R} \mid B = [a, b) \text{ for some } a, b \in \mathbb{R}\}$ where $[a, b) = \{x \in \mathbb{R} \mid a \leq x < b\}$. Let

$$\mathcal{T} = \{U \subset \mathbb{R} \mid U = \bigcup_{\alpha} B_\alpha \text{ where } B_\alpha \in \mathcal{S}\}.$$

   Show that $\mathcal{T}$ is a topology on $\mathbb{R}$.

3. Determine if each of the following is a topology on $\mathbb{R}$. If it is, prove it. If it is not, explain why not.

   (a) $\mathcal{F} = \{U \subset \mathbb{R} \mid U \text{ has finitely many elements or } U = \mathbb{R}\}$

   (b) $\mathcal{G} = \{U \subset \mathbb{R} \mid U \text{ has infinitely many elements or } U = \emptyset\}$

   (c) $\mathcal{C} = \{U \subset \mathbb{R} \mid \mathbb{R} \backslash U \text{ has finitely many elements or } U = \emptyset\}$

   (d) $\mathcal{D} = \{U \subset \mathbb{R} \mid \mathbb{R} \backslash U \text{ has infinitely many elements or} U = \mathbb{R}\}$