

Problem Session Problems

Math 244

April 5, 2010

1. The *comb space* is the subspace of \mathbb{R}^2 that is the union of $[0, 1] \times \{0\}$, $\{0\} \times [0, 1]$, and $\{\frac{1}{n}\} \times [0, 1]$ for all $n = 1, 2, 3, \dots$. (There is a picture of the comb space on p. 108 of the textbook.)
 - (a) Show that the comb space is connected.
 - (b) If we remove the point $(\frac{2}{3}, 0)$ from the comb space is the resulting space still connected?
 - (c) If we remove the point $(0, 0)$ from the comb space is the resulting space still connected?
2. A space X is *locally connected* if for each $x \in X$, and each neighborhood U of x , there is a connected neighborhood V of x which is contained in U .
 - (a) Find an example of a space that is both connected and locally connected.
 - (b) Find an example of a space that is not connected, but is locally connected.
 - (c) Show that the comb space is connected, but is not locally connected.
 - (d) Show that \mathbb{Q} , the set of rational numbers as a subspace of \mathbb{R} , is neither connected nor locally connected.