

## Additional homework problems

Math 523

Due: October 19, 2011

1. Let  $I = [0, 1]$  denote the closed interval of the real line. Show that  $I/0\sim 1$  is homeomorphic to the circle,  $S^1$ .
2. Recall that  $\mathbb{RP}^2$  is defined to be the identification space  $\mathbb{R}^3 - \{0\}/\sim$  where  $\mathbf{x} \sim \lambda\mathbf{x}$  for each  $\lambda \in \mathbb{R}, \lambda \neq 0$ . Show that  $\mathbb{RP}^2$  is homeomorphic to the identification space  $S^2/\mathbf{x} \sim -\mathbf{x}$ .
3. Show that  $\mathbb{RP}^2$  is homeomorphic to the identification space of  $D^2 = \{(x, y) | x^2 + y^2 \leq 1\} \subset \mathbb{R}^2$  under the equivalence relation  $(x, y) \sim (-x, -y)$  if  $x^2 + y^2 = 1$ . (If  $x^2 + y^2 < 1$ , then  $(x, y)$  is in its own equivalence class.)