Homework

Math 523

Due: December 12, 2011

1. Let X be the "comb space" defined as the subspace of $\mathbb{R}^2$ that is the union of $I \times \{0\}$, $\{0\} \times I$, and $\{\frac{1}{n}\} \times I$ for all $n = 1, 2, 3, \ldots$.
   (a) Show that X is contractible. That is, X deformation retracts to a point.
   (b) Show that X does not deformation retract to the point $(0, 1) \in X$.
2. Group the following into homotopy equivalence classes. No justification is needed.
   (a) S^1
   (b) $S^1 \bigvee S^1$
   (c) The cylinder, $S^1 \times I$
   (d) The Möbius band
   (e) The torus, $S^1 \times S^1$
   (f) The solid torus, $D^2 \times S^1$
   (g) The torus minus one point
   (h) The torus minus two points
   (i) The Klein bottle minus a point
   (j) The Möbuis band minus a point
   (k) $\{\vec{x} \in \mathbb{R}^2 : \|\vec{x}\| > 1\}$
   (l) $\{\vec{x} \in \mathbb{R}^2 : \|\vec{x}\| \geq 1\}$
   (m) $\{\vec{x} \in \mathbb{R}^2 : \|\vec{x}\| < 1\}$
   (n) $S^1 \cup (\mathbb{R}_+ \times \{0\}) \subset \mathbb{R}^2$
   (o) $S^1 \cup (\mathbb{R}_+ \times \mathbb{R}) \subset \mathbb{R}^2$
   (p) $S^1 \cup (\mathbb{R} \times \{0\}) \subset \mathbb{R}^2$
   (q) $\mathbb{R}^2$ with the positve x-axis deleted
   (r) $\mathbb{R}^3$ with the z-axis deleted
   (s) $\mathbb{R}^3$ with the circle $\{x^2 + y^2 = 1, z = 0\}$ deleted
   (t) $\mathbb{R}^3$ with the z-axis and the circle $\{x^2 + y^2 = 1, z = 0\}$ deleted
   (u) S^3 with one circle deleted
   (v) S^3 with two linked circles deleted
   (w) S^3 with two unlinked circles deleted