Knot Concordance and Homology Cobordism Workshop

Tim Cochran, An Overview of Concordance from the point of view of the \(n\)-solvable Filtration

I will give an overview of the \(n\)-solvable filtration of the smooth knot (or string link) concordance group including the strategy and tools used to analyze it: higher-order Alexander modules, linking forms and signature defects. I will attempt to discuss what is known about this filtration, what is not known but should be knowable by present techniques; and discuss the failings of present techniques; along the way indicating open questions and possible future directions (including putting into a context and advertising up-coming talks at this conference).

Prudence Heck, Obstructing concordance in non-simply connected manifolds

We use \(L^2\)-signatures to construct concordance invariants of null-homologous, homotopically essential knots in 3-manifolds with PTFA fundamental group. We then construct an infinite family of non-concordant knots that are characteristic to a fixed knot \(J\). Our invariants are \(\rho\)-invariants of certain 3-manifolds associated to these knots, where the 3-manifolds depend on the fixed knot \(J\).

Mark Powell, A 2nd Order Algebraic Concordance Group

I’ll talk about my project to use the symmetric chain complex of the universal cover of the knot exterior to give an instant knot concordance obstruction which is closely related to the Cochran-Orr-Teichner filtration, and to define an element of a proposed 2nd order algebraic concordance group.

Peter Horn, Higher-order signature cocycles for subgroups of the mapping class group

This is joint work with Tim Cochran and Shelly Harvey. We define families of invariants for elements of the mapping class group of \(\Sigma\), a compact, orientable surface. For a characteristic subgroup \(H \triangleleft \pi_1 \Sigma\), let \(J(H)\) denote the subgroup of mapping classes that induce the identity map on \(\pi_1 \Sigma/H\). To a unitary representation \(\psi\) of \(\pi_1 \Sigma/H\), we associate a higher-order \(\rho\)-invariant, \(\rho_\psi\), and a signature 2-cocycle \(\sigma_\psi\), a generalization of the Meyer cocycle. We show that each \(\rho_\psi\) is a quasimorphism from \(J(H) \to \mathbb{R}\), and that the \(\sigma_\psi\) span an infinite rank subgroup of \(H^2_{\text{bounded}}(J(H); \mathbb{R})\).

Shelly Harvey, Towards a \(P\)-primary decomposition of the \(n\)-solvable quotients of the knot concordance group

In the 60’s, Milnor, Tristram, and Levine defined a \(p(t)\)-primary decomposition of the algebraic concordance group, which was a crucial ingredient in its classification. For each sequence of polynomials \(P = (p_1(t), \ldots)\), we define a characteristic series of groups, called the derived series localized at \(P\). Given a knot \(K\), such a sequence of polynomials arises naturally as the sequence of orders of the higher-order Alexander modules of \(K\). These new series yield filtrations of the smooth knot concordance group that refine the \((n)\)-solvable filtration. We show that each of the successive quotients of this refined filtration contains 2-torsion and elements of infinite order. More specifically, each contains a \(\mathbb{Z}_\infty^\times\) and a \(\mathbb{Z}/2^\infty\). This is joint work with Tim Cochran and Constance Leidy.

John Burke, Defining Strongly Coprime for Multivariable Laurent Polynomials and Further Structure of the \(n\)-solvable Filtration

In this talk, we will define what it means for multivariable Laurent polynomials to be strongly coprime. This will then allow us to construct a knot, using genetic infection with string links, which is of infinite order in the group, \(\mathcal{G}\), of 2-solvable knots modulo 2.5-solvable knots and is linearly independent from all previous knots in \(\mathcal{G}\) constructed by Cochran, Harvey, and Leidy.
Cornelia Van Cott, *Concordance of Bing doubles and boundary genus*

The construction of Bing doubling and its relationship to knot concordance has been the subject of investigation in recent years. In this talk, we will show that for most knots $K$, the iterated Bing doubles of $K$ are in fact very far from being slice. In particular, suppose that $K$ has nontrivial signature $\sigma$. Then, if the components of the $n$th-iterated Bing double of $K$ bound disjoint surfaces in $B^4$, the genus of each of these surfaces is at least $2^{n-1}\sigma$. The same result holds with $\sigma$ replaced by $2\tau$, twice the Ozsváth-Szabó knot concordance invariant. This is joint work with Chuck Livingston.

Bridget Franklin, *The Effect of Infecting Curves on Knot Concordance*

It is known that the $n$-solvable filtration of the knot concordance group, $\mathfrak{F}$, has infinite rank in each successive quotient $\mathfrak{F}/\mathfrak{F}_n$ for every $n \geq 0$. These families of linearly independent knots are produced by Cochran, Harvey, and Leidy using iterated satellite operations, denoted by $K_i = R_i(\eta, K_{i-1})$. They first produced a linearly independent set by varying the classical signatures of the base knot $K_0$. Later, by taking $\{R_i\}$ to be $n$ robust doubling operators with varying strongly coprime Alexander polynomials, they produced $n$ distinct linearly independent sets in each quotient $\mathfrak{F}/\mathfrak{F}_n$. In this talk, we show that varying the infecting curve, $\eta \subset S^3 \setminus R_2$, produces another distinct linearly independent set of $K_2 = R_2(\eta, R_1(K_0))$ in $\mathfrak{F}/\mathfrak{F}_2$ which appears even while fixing $R_2, R_1$ and the base knot $K_0$.

Stefan Friedl, *Criteria for sliceness*

In 1981 Mike Freedman showed that knots with Alexander polynomial one are topologically slice. This implies for example that Whitehead doubles of knots are topologically slice. I will talk about results with Peter Teichner and Tim Cochran which generalize Freedman’s results.

Matt Hedden, *A survey of smooth concordance techniques*

I’ll give an overview of the tools and invariants within concordance which are unique to the smooth group; that is, those methods which can distinguish topologically concordant knots. While I will try to pay homage to most of the existing techniques, the heart of the talk will be on the tools that come from Ozsváth-Szabó theory. There are many concordance invariants coming from this package alone, and I will indicate what tools exist, what has been proved with them, and what one can reasonably hope to prove within the existing theory.

Adam Levine, *Bordered Heegaard Floer homology and knot doubling operators*

We show how to use bordered Heegaard Floer homology to compute the Ozsváth-Szabó $\tau$ invariant for a family of knots that generalizes Whitehead doubles, providing a hands-on introduction to the bordered theory. As an application, we show that if $K$ is any knot with $\tau(K) > 0$, then the all-positive Whitehead double of any iterated Bing double of $K$ is topologically but not smoothly slice.

Taehee Kim, *The cobordism group of homology cylinders and combinatorial torsions*

A homology cylinder is a homology cobordism over a surface with fixed markings for its boundary. The homology cobordism classes of homology cylinders over a fixed surface form a group, which can be regarded as an enlargement of the mapping class group. Using torsion invariants, we show that the abelianization of this group is infinitely generated if the first Betti number of the surface is positive and has infinite rank if the surface has more than one boundary component. We also show similar results for the homology cylinder analogue of the Torelli group. This is joint work with Jae Choon Cha and Stefan Friedl.
Carolyn Otto, *The (n)-Solvable Filtration of the Link Concordance Group and Milnor’s Invariants*

We will announce several new results about the $(n)$-solvable filtration of the string link concordance group, denoted $F_n$. First, we will establish a relationship between $(n)$-solvability of a link and its Milnor’s $ar{\mu}$-invariants. Using this, we show the “other half” of the filtration, $F_{n+1}$, is nontrivial for links with sufficiently many components. We also show that links modulo 1-solvability is a non-abelian group.

Shea Vela-Vick, *Pontryagin invariants and integral formulas for Milnor’s triple linking number*

To each three-component link in the 3-dimensional sphere we associate a characteristic map from the 3-torus to the 2-sphere, and establish a correspondence between the pairwise and Milnor triple linking numbers of the link and the Pontryagin invariants that classify its characteristic map up to homotopy. This can be viewed as a natural extension of the familiar fact that the linking number of a two-component link is the degree of its associated Gauss map from the 2-torus to the 2-sphere. When the pairwise linking numbers are all zero, we give an integral formula for the triple linking number analogous to the Gauss integral for the pairwise linking numbers. The integrand in this formula is geometrically natural in the sense that it is invariant under orientation-preserving rigid motions of the 3-sphere.

Kent Orr, *Homology cobordism, $L^2$ methods, and amenable groups*

$L^2$ signatures play a central role in the study of knot concordance, a classical relation on knots first defined and studied by Fox and Milnor, and closely allied with deep considerations in the study of stratified space and the classification of 4-manifolds. In collaboration with Jae Choon Cha, and using a new approach which subsumes past results, we significantly extend key results concerning invariance of $L^2$ signatures and betti numbers.

Jae Choon Cha, *Amenable $L^2$-theoretic methods and applications*

We discuss some applications of $L^2$-theoretic methods for homology with coefficients in group von Neumann algebras over amenable groups. We focus on the ability to use groups with torsion. In particular, we exhibit examples of exotic homology cobordism classes from “hidden torsion” in the localization of 3-manifold groups, and a large family of concordance classes of knots which are not distinguished by the $L^2$-signature obstructions of Cochran-Orr-Teichner and invariants of Casson-Gordon and Levine. This work is partly joint with Kent Orr.

Jen Hom, *The knot Floer complex, cabling and concordance*

We will use bordered Heegaard Floer homology to give a formula for tau of the $(p, q)$-cable of a knot $K$ in terms of $p$, $q$, and two smooth concordance invariants, tau and epsilon, associated to the knot Floer complex of K. We will also describe the behavior of epsilon under cabling, allowing us to compute tau of iterated cables. As a consequence, we will show that for any integer $n$, there exist knots $K$ and $K'$ with tau($K$)=tau($K'$)=n such that tau of the $(p, q)$-cables of $K$ and $K'$ are not equal for any pair of relatively prime integers $p$ and $q$. Finally, we will discuss some of the properties of epsilon; in particular, epsilon is strictly stronger than tau in determining obstructions to a knot being slice.
Chris Davis, *Localized metabelian ρ invariants as obstructions to torsion in the knot concordance group*

I will define a family of metabelian von Neumann ρ invariants corresponding to localizations of the rational Alexander module of a knot. While these are not well defined on the concordance group, they still provide obstructions to a knot’s being finite order. I derive a computable bound on these invariants. I apply this to give a proof that a new infinite family of twist knots is linearly independent in the topological concordance group.

Matt Hedden, *Instantons, concordance, and Whitehead doubling*

I’ll discuss some joint work Paul Kirk which shows that untwisted Whitehead doubles generate an infinite rank subgroup of the smooth concordance group (which is, of course, rank zero in the topological concordance group). The proof uses a technique from gauge theory, which can be viewed as an interplay between the construction of non-empty moduli spaces of SO(3) instantons over 4-manifolds and the calculation of Chern-Simons invariants of flat SO(3) connections on 3-manifolds.

Danny Ruberman, *Slice Knots and the Alexander Polynomial*

It has been known since the early 1980’s that there are knots that are topologically (flat) slice, but not smoothly slice. These result from Freedman’s proof that knots with trivial Alexander polynomial are topologically slice, combined with gauge-theory techniques originating with Donaldson. In joint work with C. Livingston and M. Hedden, we answer the natural question of whether Freedman’s result yields all topologically slice knots. We show that the group of topologically slice knots, modulo those with trivial Alexander polynomial, is infinitely generated. The proof uses Heegaard-Floer theory. Some applications to link concordance will be discussed, time permitting.

Se-Goo Kim, *Splitting property of von Neumann ρ-invariants of knots*

We give a sufficient condition for the connected sum of two given knots under which the two knots have vanishing von Neumann ρ-invariants if their connected sum does. The condition is described in terms of metabolizers of Cochran-Orr-Teichner’s higher-order linking forms which includes primeness of Alexander polynomials. We then present an example as an application. This is joint work with Taehee Kim.

Kate Kearney, *Concordance Genus of Knots*

The concordance genus of a knot is the least three–genus of a knot concordant to the given knot. In this talk, we will discuss its relationship to other invariants and techniques for calculating the concordance genus. This will be illustrated with examples of recent calculations of concordance genus, including 11–crossing knots and sums of torus knots.

Tim Cochran, *Filtering Smooth Concordance classes of Topologically Slice Knots*

The n-solvable filtration of the smooth knot (or string link) concordance group, C, has provided a framework for many advances in the study of knot concordance. However it is useless for studying the subgroup, T, of topologically slice knots. We define and investigate new highly non-trivial filtrations of C and give evidence (but don’t prove) that they induce non-trivial filtrations of T. These are essentially refinements of \( \{F_n\} \). We first define natural monoid filtrations of C, called the n-positive filtration, and n-negative filtration, that are much more amenable to gauge-theoretic techniques. Their intersection, \( \{NP_n\} \), is a filtration by subgroups. Currently-defined gauge-theoretic invariants seem to vanish on \( NP_0 \), yet we show that this filtration is as highly non-trivial as \( \{F_n\} \). At present, we are only able to show that the induced filtration of T is non-trivial (for all n) under an extra “weak homotopy ribbon” assumption.